

Concepts

Signed Fractions, Equivalent Fractions,
Division of Signed Fractions, Writing
Fractions in Lowest Terms, Divisibility
Rules, Complex Fractions

Introduction to Signed Fractions

- Fractions consist of two parts: numerator and denominator

$$\frac{2}{3} = \frac{\text{numerator}}{\text{denominator}}$$

- There are two types of fractions: proper and improper

$$\frac{4}{7} = \text{proper}$$

$$\frac{5}{3} = \text{improper}$$

Equivalent Fractions

- Equivalent fractions are equal fractions.

$$\frac{2}{3} = \frac{4}{6}$$

- To create an equivalent fraction, multiply both the numerator and denominator of the fraction by the same number:

$$\frac{6}{7} = \frac{6 \cdot 2}{7 \cdot 2} = \frac{12}{14} = \frac{6}{7}$$

Division of Signed Fractions

- Follow the sign rules from previous chapters.
- Remember the phrase:
 - **Keep**, **change**, **flip**

$$\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \cdot \frac{2}{1} = \frac{4}{3} \text{ or } 1\frac{1}{3}$$

The diagram shows the equation $\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \cdot \frac{2}{1} = \frac{4}{3} \text{ or } 1\frac{1}{3}$. Three arrows originate from the bottom: a red arrow labeled 'Keep' points to the first fraction $\frac{2}{3}$; a blue arrow labeled 'Change' points to the division sign \div ; and a purple arrow labeled 'Flip' points to the second fraction $\frac{1}{2}$. Additional arrows show the transformation of the second fraction to $\frac{2}{1}$ and the final result $\frac{4}{3}$.

Keep the first number, **change** division to multiplication, and **flip** the next number. Multiply straight across or reduce if possible.

Writing Fractions in Lowest Terms

- A fraction in lowest terms has no common factors between the numerator and denominator.
- There are different methods to use to put a fraction into lowest terms:
 - Use divisibility rules (dividing by a common factor)
 - Prime factorization
 - Composite numbers

Divisibility Rules

- 2, 5, and 10
 - A number is divisible by 2 if and only if its ones digit is 0, 2, 4, 6, or 8.
 - A number is divisible by 5 if and only if its ones digit is 0 or 5.
 - A number is divisible by 10 if and only if its ones digit is 0.
- 4 and 8
 - A number is divisible by 4 if and only if the number represented by its last two digits is divisible by 4.
 - A number is divisible by 8 if and only if the number represented by its last three digits is divisible by 8.

Divisibility Rules (cont'd)

■ 3 and 9

- A number is divisible by 3 if and only if the sum of its digits is divisible by 3.
- A number is divisible by 9 if and only if the sum of its digits is divisible by 9.

■ 11

- A number is divisible by 11 if and only if 11 divides the difference of the sum of the digits whose place values are odd powers of 10 and the sum of the digits whose place values are even powers of 10.

■ 6

- A number is divisible by 6 if and only if both of the tests for divisibility by 2 and 3 hold.

Reduce $\frac{124}{340}$ using different methods

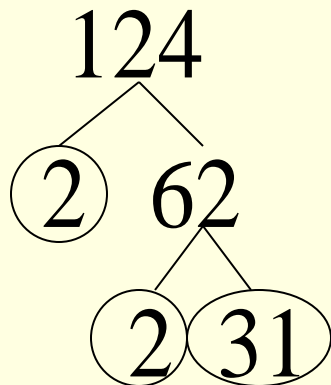
- Since both numbers end in a number divisible by 2, we can reduce by 2:

$$\frac{124}{340} = \frac{62}{170} = \frac{31}{85}$$

- You can use the same divisibility rule several times with the same problem.
- This problem also follows the divisibility rule for 4.

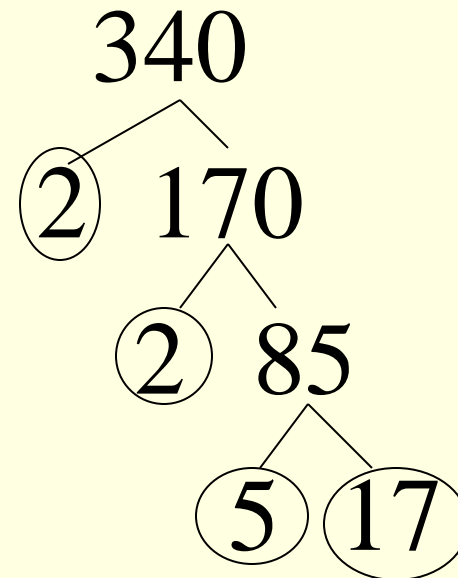
Reduce $\frac{124}{340}$ using different methods

- Prime factorization – list each number as a product of their primes



$$124 = 2 \times 2 \times 31$$

$$340 = 2 \times 2 \times 5 \times 17$$



$$\frac{124}{340} = \frac{31}{5 \cdot 17} = \frac{31}{85}$$

Composite Numbers $\frac{124}{340}$

- Composite numbers are two numbers, other than 1 and the number, that are multiplied together to give you a number. For instance, $12 = 3 \times 4$ and $36 = 6 \times 6$

$$\frac{124}{340} = \frac{4 \times 31}{4 \times 85} = \frac{31}{85}$$

- Each method that we used brought us to the same answer.

Complex Fractions

- A complex fraction is a fraction written over another fraction:

$$\frac{\frac{2}{3}}{\frac{5}{7}}$$

- Another way of writing the complex fraction above is:

$$\frac{2}{3} \div \frac{5}{7}$$

You would follow the rules for dividing fractions.

Fractions and Exponents

- The same rules apply for fractions when followed by an exponent:

$$\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{16}$$

$$\left(-\frac{2}{3}\right)^3 \left(\frac{1}{2}\right)^2 = \left(-\cancel{\frac{2}{3}}\right)\left(-\cancel{\frac{2}{3}}\right)\left(-\frac{2}{3}\right)\left(\cancel{\frac{1}{2}}\right)\left(\cancel{\frac{1}{2}}\right) = -\frac{2}{27}$$

Order of Operations

- The Order of Operations – PEMDAS – will always be followed no matter what.

$$\frac{8}{3} \left(\frac{1}{4} - \frac{1}{2} \right)^2 = \frac{8}{3} \left(\frac{1}{4} - \frac{2}{4} \right)^2 = \frac{8}{3} \left(-\frac{1}{4} \right)^2$$

$$\frac{8}{3} \left(\frac{1}{4} \right) \left(\frac{1}{4} \right) = \frac{8}{48} = \frac{\cancel{8} \cdot 1}{\cancel{8} \cdot 6} = \frac{1}{6}$$