

# Factoring Guidelines

- Greatest Common Factor
- Two Terms
- Three Terms
- Four Terms

# Greatest Common Factor

## Factoring by Finding the Greatest Common Factor

- Always check for the ***Greatest Common Factor*** first.
- Are there any common factors? If so, factor out the common factor.

$$6x^2 + 3x + 12 = 3(2x^2 + x + 4)$$

# Special Two Term Factoring

# Polynomials With Two Terms

- If there are two terms, decide if one of the following could be applied.
  - Difference of two squares: If in this form  $a^2 - b^2$  then  $(a - b)(a + b)$  will be the form of the answer.
  - Difference of two cubes: If in this form  $a^3 - b^3$  then  $(a - b)(a^2 + ab + b^2)$  will be the form of the answer.
  - Sum of two cubes: If in this form  $a^3 + b^3$  then  $(a + b)(a^2 - ab + b^2)$  will be the form of the answer.

# Difference of Two Squares

$$4x^2 - 16$$

$$4(x^2 - 4) \quad \text{Factor out the GCF first}$$

$$4(x - 2)(x + 2)$$

After factoring out the GCF, check your answer to see if it can be factored again. The problem can be factored again, so follow the procedure for finding the “difference of squares.”

$$4x^2 - 49$$

$$(2x + 7)(2x - 7)$$

# Difference of Two Cubes

$$a^3 - b^3$$

$$(a - b)(a^2 + ab + b^2)$$

This is the form that the problem would be in followed by the form the answer will be in.

$$8x^3 - 27$$

$$a = 2x$$

$$b = 3$$

$$(2x - 3)(4x^2 + 6x + 9)$$

First find the cube root of each term. That gives you the  $a$  and  $b$  that you need to plug into the form above.

# Difference of Two Cubes

$$a^3 - b^3$$

$$(a - b)(a^2 - ab + b^2)$$

This is the form that the problem would be in followed by the form the answer will be in.

$$27x^3 - 512$$

$$a = 3x$$

$$b = 8$$

$$(3x - 8)(9x^2 - 24x + 64)$$

First find the cube root of each term. That gives you the  $a$  and  $b$  that you need to plug into the form above.



# Factoring Three Terms

# Factor $x^2 + bx + c$

- To factor a trinomial of the form  $x^2 + bx + c$ , look for two numbers whose product is  $c$  and whose sum is  $b$ . The factored form is:

$$(x + \text{one number})(x + \text{other number})$$

- Example:  $x^2 + 7x + 12$
- The factors of 12 are:

(1, 12), (2, 6), and (3, 4).

The factors when added together that give me 7 are (3, 4). Therefore, the factored form is:

$$(x + 3)(x + 4)$$

# Factor $ax^2 + bx + c$

- To factor a trinomial of the form  $ax^2 + bx + c$ , multiply  $a \cdot c$  and list all the factors of that number. Then build a “box” and place the  $ax^2$  term in the top right corner and the  $c$  term in the bottom left (see next slide).
- Choose the factors that will give you the middle term and place them in the other spaces in the box. You must also choose the signs for the middle terms.

# Signs

- If the **middle term is negative** and the **last term is positive**, both terms must be negative.
- If the **middle term is positive** and the **last term is positive**, both terms must be positive.
- If the **middle term is negative** and the **last term is negative**, one term will be positive and the other term will be negative.
- You will need to decide which term needs to be negative or positive by checking your answer.
- Then look at each row and each column and list the common factors. The common factors will be your factored answer.

# First step...

Example:

$$4x^2 - 19x + 12$$

Common Factors		
	$4x^2$	$x$
	$x$	$12$

First, place the first term in the middle box and the last term in last box.

# Second step...

Example:

$$4x^2 - 19x + 12$$

Common Factors		
	$4x^2$	$3x$
	$16x$	$12$

Multiply  $4(12)$  to get 48.  
Now list all the factors of 48.

$(1, 48)$ ,  $(2, 24)$ ,  $(3, 16)$ ,  
 $(4, 12)$ ,  $(6, 8)$ .

Since the factor  $(3, 16)$  gives me 19 for the middle term, place those factors in the remaining empty spaces, remembering that they are "x" terms.

# Second step...

Example:

$$4x^2 - 19x + 12$$

Common Factors	4x	3
x	4x <sup>2</sup>	3x
4	16x	12

Now look at each row and each column and determine what the common factors are and write them in the green box.

4x<sup>2</sup> and 16x have a common factor of 4x.

3x and 12 have 3 in common.

4x<sup>2</sup> and 3x have x in common.

16x and 12 have 4 in common.

# Determining the Signs

Common Factors	4x	3
x	4x <sup>2</sup>	3x
4	16x	12

$$(4x \quad 3)(x \quad 4)$$

We just need to determine the signs that go into the problem. Since the original problem had the middle term as negative and the last term as positive, then looking back at our “rules”, both terms will be negative.

Final answer is  $(4x-3)(x-4)$



# Factoring Four Terms

# Factor By Grouping

- **If there are four or more terms, try to factor by grouping.**

$$x^3 + 3x^2 - 5x - 15$$

$$(x^3 + 3x^2) - (5x + 15)$$

$$x^2(x + 3) - 5(x + 3)$$

$$(x^2 - 5)(x + 3)$$

First, group the first two terms together and the last two terms together. Factor out the common factor in each group which becomes your first binomial. When you factored out the common factors, you created the same binomial  $(x + 3)$  which becomes your second factor.

# Remember...

- Always check to see if any factors in the *factored* polynomial can be factored further.
- Always check your answers by multiplying (in most cases using FOIL) to make sure you get what you started with!