Adding and Subtracting Rational Expressions

A rational expression is a fraction. The same rules apply for adding and subtracting rational expressions as they do fractions – common denominators are needed.

Finding Common Denominators

$$\frac{3}{8}+\frac{1}{6}=\frac{3}{8}\left(\frac{3}{3}\right)+\frac{1}{6}\left(\frac{4}{4}\right)=\frac{9}{24}+\frac{4}{24}=\frac{13}{24}$$

The common denominator for 8 and 6 is 24. Multiply the first fraction by 3/3 and the second fraction by 4/4 to get the common denominator. Now add the fractions together and reduce the answer if possible.

$$\frac{a}{4}-\frac{2a}{8}=\frac{a}{4}\left(\frac{2}{2}\right)-\frac{2a}{8}=\frac{2a}{8}-\frac{2a}{8}=0$$

If the rational expressions have the same denominator, then add or subtract the numerators together and simplify. If the numerator is factorable, it needs to be factored.

$$\frac{3x^2+2x}{x-1}-\frac{10x-5}{x-1}$$

Since the fractions have a common denominator, we can combine the numerators by subtracting the binomial 10x + 5 (don't forget to change the signs). The answer is factorable, so factor the answer and then cancel any like terms in the numerator and denominator.

$$\frac{3x^2+2x-(10x+5)}{x-1}$$

$$\frac{3x^2-8x+5}{x-1}=\frac{\cancel{(x-1)}(3x-5)}{\cancel{x-1}}=3x-5$$

That problem is pretty straight forward. What about a problem that doesn't have common denominators? If that is the case, we need to find the common denominator. Let's look at just the denominators for a problem and determine what the common denominator should be.

Denominators: $x+1$, $2x+2$ Factor the denominators: $x+1$, $2(x+1)$

The common denominator will be made up of any missing factors when comparing the two denominators together. The common denominator will be 2(x + 1). We need to create equivalent fractions with the common denominator. Here is the entire problem.

Problem → Factored Form → Multiply by Missing Factor → Add → Reduce

$$\frac{6}{x+1}+\frac{10}{2x+2}=\frac{6}{x+1}+\frac{10}{2(x+1)}=\frac{6(2)}{x+1(2)}+\frac{10}{2(x+1)}=\frac{12+10}{2(x+1)}=\frac{\cancel{22}^{11}}{\cancel{2}(x+1)}=\frac{11}{x+1}$$

$$\frac{x+8}{x^2-5x-6}+\frac{x+1}{x^2-4x-5}=\frac{x+8}{(x-6)(x+1)}+\frac{x+1}{(x-5)(x+1)}$$

a b c b

Common denominator is made up of abc: $(x+1)(x-6)(x-5)$. Now multiply both the numerator and the denominator by what is missing in each denominator.

$$\frac{(x+8)(x-5)}{(x-6)(x+1)(x-5)}+\frac{(x+1)(x-6)}{(x-5)(x+1)(x-6)}=\frac{x^2+3x-40+x^2-5x-6}{(x-6)(x+1)(x-5)}=$$

$$\frac{2x^2-2x-46}{(x-6)(x+1)(x-5)}=\frac{2(x^2-x-23)}{(x-6)(x+1)(x-5)}$$

$$\frac{9x}{x-10}-\frac{x}{x-3}=\frac{9x(x-3)-x(x-10)}{(x-3)(x-10)}=\frac{9x^2-27x-x^2+10x}{(x-3)(x-10)}=\frac{8x^2-17x}{(x-3)(x-10)}=$$

$$\frac{x(8x-17)}{(x-3)(x-10)}$$

$$\frac{10x}{x^2-9}-\frac{5}{x+3}=\frac{10x}{(x+3)(x-3)}-\frac{5}{x+3}=\frac{10x}{(x+3)(x-3)}-\frac{5(x-3)}{(x+3)(x-3)}=$$

$$\frac{10x-5(x-3)}{(x+3)(x-3)}=\frac{10x-5x+15}{(x+3)(x-3)}=\frac{5x+15}{(x+3)(x-3)}=\frac{5\cancel{(x+3)}}{\cancel{(x+3)}(x-3)}=\frac{5}{(x-3)}$$