Basics of Graphing

The graphing that we will be doing in this class covers the most basic geometric form – a line. The lines we will be graphing on the Cartesian or Rectangular Coordinate System will all be linear lines (straight lines). Here is a picture of the rectangular coordinate system:

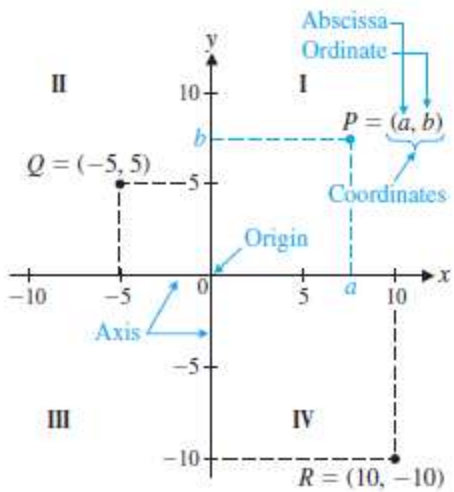


Figure 1 The Cartesian (rectangular) coordinate system

Notice that where the x- and y-axis meet is called the *origin* and it has a coordinate of (0, 0). The system is broken up into four quadrants that are named in a counter-clockwise rotation starting in the upper right corner. They are labeled I, II, III, and IV. They are referred to as "regions."

Linear Equations in Two Variables

DEFINITION Linear Equations in Two Variables

A **linear equation in two variables** is an equation that can be written in the **standard form**

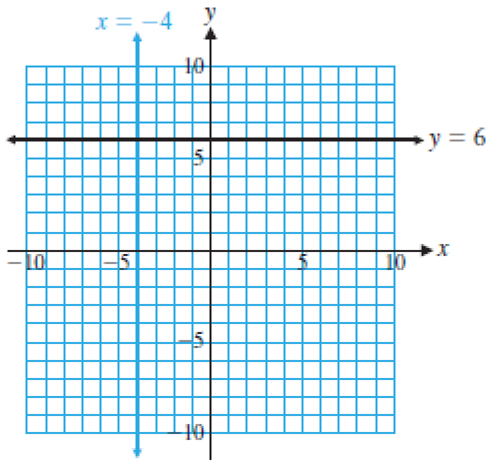
$$Ax + By = C$$

where A, B, and C are constants (A and B not both 0), and x and y are variables.

The solution to a linear equation in two variables is a coordinate (x, y) that satisfies the equation. The **solution set** of an equation in two variables is the set of all solutions of the equation. The **graph** of an equation is the graph of its solution set.

Horizontal and Vertical Lines

A *horizontal line* has an equation of $y = c$ and a *vertical line* has an equation of $x = c$. A horizontal line has slope equal to zero and a vertical line has slope that is undefined.



Slope of a Line

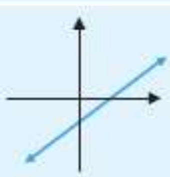
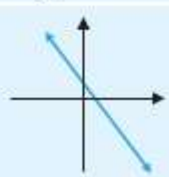
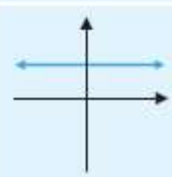
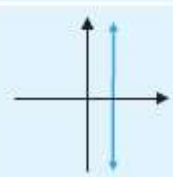
DEFINITION Slope of a Line

If a line passes through two distinct points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, then its slope is given by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 \neq x_2$$

$$= \frac{\text{vertical change (rise)}}{\text{horizontal change (run)}}$$

Table 1 Geometric Interpretation of Slope

Line	Rising as x moves from left to right	Falling as x moves from left to right	Horizontal	Vertical
Slope	Positive	Negative	0	Not defined
Example				

Finding Equations of a Line

$$y = mx + b$$

$$\uparrow \qquad \nwarrow$$

slope y-intercept

If an equation is in the form of *y* = *mx* + *b,* you can look at the equation to determine the slope and the y-intercept. Examples:

y = mx + b	m = slope	b = y-intercept (0, b)
y = 4x – 6	m = 4	b = (0, -6)
y = 2/3x + 5	m = 2/3	b = (0, 5)

If given the slope and the y-intercept and asked to find the equation of the line, just plug in the known information into the *y* = *mx* + *b* form. Example:

m = 3, y-intercept (0, 7)

Equation of the line is y = 3x + 7

When asked to find the equation of the line and are given two points, first you must find the slope of the line. The formula for finding the slope of a line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(x_1, y_1) \quad (x_2, y_2)$$

Labeling your points ahead of time will help with substituting into the slope formula. It does not matter which one is point 1 and which is point 2. As long as you use them in the same order, you will get the same answer.

Given: (2, 5) and (-2, 7)

$\frac{7-5}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$ Which means slope is -1/2 (in lowest terms)

So we now know the slope of the line. Use the slope and one of the given points (it does not matter which one) to find *b* (y-intercept). I'm going to use the point (2, 5) and slope of -1/2 (found above).

5 = -1/2(2) + b (I substituted the known information into the form $y = mx + b$)
5 = -1 + b
6 = b (I solved to find b.)

Equation of the line is: y = -1/2x + 6

Another formula that you can use is:

$$y - y_1 = m(x - x_1).$$

This formula is called the *point-slope* formula and can be used when the slope and a point are given.

Example:

Given: *m* = 3 and (5, -4)

y – (-4) = 3(x – 5)
y + 4 = 3x – 15
y = 3x – 19

When the line is graphed, this is what it looks like. The slope of the line is positive so the line moves upwards. Notice that the point on the graph (0, -19) is the y-intercept we found when we used the point-slope formula.