Completing the Square

When the principle of zero products and the principle of square roots do not yield the exact zeros of a function, we can use a procedure called **completing the square** and then we can use the principle of square roots.

To solve a quadratic equation by completing the square:

1. Isolate the terms with variables on one side of the equation and arrange them in descending order.
2. Divide by the coefficient of the squared term if that coefficient is not 1.
3. Complete the square by taking half the coefficient of the first-degree term and adding its square on both sides of the equation.
4. Express one side of the equation as the square of a binomial.
5. Use the principle of square roots.
6. Solve for the variable.

Example:

$$x^2 + 6x = 7$$
$$x^2 + 6x \qquad = 7$$
$$x^2 + 6x + 9 = 7 + 9$$

Divide 6 by 2 to get 3 and then square the 3 to get 9. Add 9 to both sides of the equation.

$$x^2 + 6x + 9 = 7 + 9$$
$$(x+3)^2 = 16$$

The number you get when you divide the middle term by 2 is the number to use to express one side of the equation as the **square of a binomial.**

$$x^2 + 6x + 9 = 7 + 9$$
$$(x+3)^2 = 16$$
$$\sqrt{(x+3)^2} = \sqrt{16}$$
$$x + 3 = \pm 4$$
$$x = -3 \pm 4$$
$$x = -3 + 4 \text{ and } -3 - 4$$
$$x = 1, -7$$

Now that we have a square of a binomial, we can use the principle of square roots to solve the equation. Take the square root of both sides and solve for the variable.

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