

Exponential and Logarithmic Functions

Algebra of Functions

Sum $(f + g)(x) = f(x) + g(x)$
Difference $(f - g)(x) = f(x) - g(x)$
Product $(f \bullet g)(x) = f(x) \bullet g(x)$
Quotient $(\frac{f}{g})(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$

Composite Functions

The notation $(f \circ g)(x)$ means “ f composed with g ”

$$(f \circ g)(x) = f(g(x))$$
$$(g \circ f)(x) = g(f(x))$$

If f is a function, then f is a one-to-one function only if each y -value (output) corresponds to only one x -value (input).

Horizontal Line Test

If every horizontal line intersects the graph of a function at most once, then the function is a one-to-one function.

The **inverse** of a one-to-one function f is the one-to-one function f^{-1} that is the set of all ordered pairs (b, a) such that (a, b) belongs to f .

To find the inverse of a one-to-one function $f(x)$:

1. Replace $f(x)$ with y .
2. Interchange x and y .
3. Solve for y .
4. Replace y with $f^{-1}(x)$.

A function of the form $f(x)=b^x$ is an exponential function, where $b > 0$, $b \neq 1$, and x is a real number.

Uniqueness of b^x : If $b > 0$ and $b \neq 1$, then $b^x = b^y$ is equivalent to $x = y$.

Logarithmic definition:

If $b > 0$ and $b \neq 1$, then

$$y = \log_b x \text{ means } x = b^y$$

For positive x and real number y .

Properties of logarithms:

If b is a real number, $b > 0$ and $b \neq 1$, then

$$\log_b 1 = 0$$

$$\log_b b^x = x$$

$$b^{\log_b x} = x$$

Logarithmic function:

If $B > 0$ and $b \neq 1$, then a **logarithmic function** is a function that can be defined as

$$f(x) = \log_b x$$

The domain of f is the set of positive real numbers, and the range f is the set of real numbers.

Let x , y , and b be positive numbers and $b \neq 1$.

Properties of Logarithms

Product property:

$$\log_b xy = \log_b x + \log_b y$$

Quotient property:

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

Power property:

$$\log_b x^r = r \log_b x$$

Common logarithms:

$\log x$ means $\log_{10} x$

Natural logarithms:

$\ln x$ means $\log_e x$

**Continuously-compounded
interest formula:**

$$A = Pe^{rt}$$

where r is the annual interest rate
for P dollars invested for t years.

Logarithm property of equality:

Let $\log_b a$ and $\log_b c$ be real numbers
and $b \neq 1$. Then

$\log_b a = \log_b c$ is equivalent to $a = c$.