Finding Inverse Functions

In mathematics, being able to "undo" a problem plays a critical role in the solution process of some problems. In other words, we want to find the ***inverse*** of the function.

Inverse of a Relation

A relation is a set of ordered pairs and its inverse would be an exchange of the x and y terms in the relation.

$R = \{(-4, 2), (3, 2), (0, -1), (3, -2)\}$

$R^{-1} = \{(2, -4), (2, 3), (-1, 0), (-2, 3)\}$

The x and y values were simply exchanged.

Horizontal Line Test

We have used the *vertical line test* to determine if a relation is a function. If a vertical line is drawn through the graph, it cannot touch the graph at more than one point on the graph. The ***horizontal line test*** helps us determine if a function has an inverse.

THEOREM

Let *f* be a function. We say that the graph of *f* passes the ***horizontal line test*** if every horizontal line in the plane intersects the graph no more than once. If *f* passes the *horizontal line test*, the f^{-1} is also a function.

In summary, the inverse of f^{-1} of a function *f* is also a function if and only if *f* is one-to-one and *f* is one-to-one if and only if its graph passes the horizontal line test.

To find an inverse function:

1. Replace *f(x)* with the variable y.

$$f(x) = 5x - 4$$
$$y = 5x - 4$$

2. Interchange *x* and *y* in the equation.

$$x = 5y - 4$$

3. Solve for y.

$$x = 5y - 4$$
$$x - 4 = 5y$$
$$\frac{x-4}{5} = y$$

4. Replace the *y* in the equation to $f^{-1}(x)$.

Some examples:

$$f(x) = x^2 + 1$$
$$y = x^2 + 1$$
$$x = y^2 + 1$$
$$x - 1 = y^2$$
$$\sqrt{x-1} = y$$

$$r(x) = \frac{x-1}{3x+2}$$
$$y = \frac{x-1}{3x+2}$$
$$x = \frac{y-1}{3y+2}$$
$$x(3y+2) = y - 1$$
$$3xy + 2x = y - 1$$
$$3xy - y = -2x - 1$$
$$y(3x-1) = -2x - 1$$
$$y = \frac{-2x-1}{3x-1}$$
$$f^{-1}(x) = \frac{-2x-1}{3x-1}$$

$$f(x) = \sqrt{x-4}$$
$$y = \sqrt{x-4}$$
$$x = \sqrt{y-4}$$
$$(x)^2 = (\sqrt{y-4})^2$$
$$x^2 = y - 4$$
$$x^2 + 4 = y$$
$$x^2 + 4 = f^{-1}(x)$$