Linear Regression and Modeling

Regression, in general, is the problem of estimating a conditional expected value. Linear regression is called "linear" because the relation of the dependent to the independent variables is a linear function of some parameters. Linear regression can be used in mathematical modeling to give us estimates of future data.

Mathematical modeling can be done in three steps:

1. ***Construct*** the mathematical model.
2. ***Solve*** the mathematical model.
3. ***Interpret*** the mathematical model.

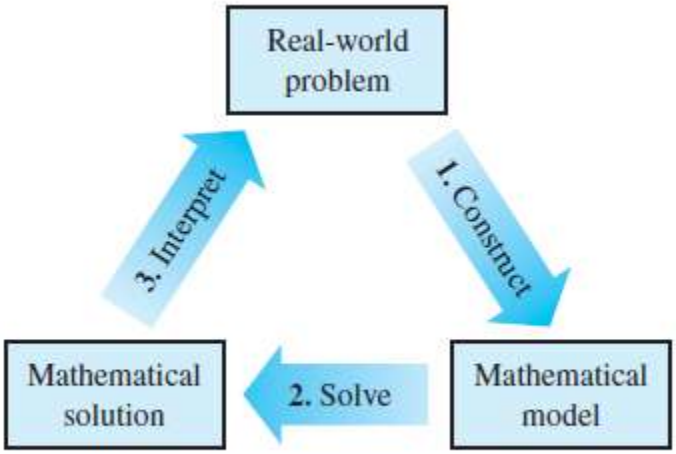

Figure 1

Rockswold, G. K., &; Krieger, T. A. (2012). *Beginning and intermediate algebra with applications and visualization*. (3rd.). Boston, MA: Addison-Wesley.

Slope as a Rate of Change

If x and y are related by the equation where *m* and *b* are constraints, and $m \neq 0$, then x and y are ***linearly related.*** To find the rate of change (slope) we use the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Change in } y}{\text{Change in } x}$$

First, an example to show you what the equation of the line can tell us.

Estimating body surface area. The equation $a = 28.55w + 118.7$ expresses body surface area (BSA) for felines in terms of weight, where *a* is BSA in square inches and *w* is weight in pounds.

(A) Interpret the slope of the BSA equation.

(B) What is the effect of a one-pound increase in weight?

Explanation:

(A) The slope in the equation is 28.55. This is the rate-of-change of BSA with respect to weight in square inches per pound.

(B) Since slope is the ratio of rise to run, increasing *w* by 1 pound (run) increasing *a* by 28.55 square inches (rise).

Now an example:

A 400-pound load of grain is dropped from an altitude of 2,880 feet and lands 80 seconds later.

(A) Find a linear model relating altitude *a* (in feet) and time in the air *t* (in seconds).

(B) How fast is the cargo moving when it lands?

If $a = mt + b$ is the linear equation relating altitude *a* and time in air *t*, then the graph of this equation must pass through the following points:

$(t_1, a_1) = (0, 2880)$ Cargo is dropped from the plane
$(t_2, a_2) = (80, 0)$ Cargo hits the ground

First we find the slope of the line:

$$m = \frac{0 - 2880}{80 - 0} = \frac{-2880}{80} = -36$$

Next we use the point-slope formula to find the linear model relating altitude *a* (in feet) and time in the air *t* (in seconds):

$y - y_1 = m(x - x_1)$

$y - 0 = -36(x - 80)$
$y = -36x + 2880$

The rate of decent of the grain is -36 feet per second. Since this is a distance, we use the absolute value of the feet per second or |-36| = 36 feet per second.