Multiplying and Dividing Rational Expressions

Factor First

See the Factoring Guidelines file under Basic Algebra (www.mathrules.net)

- Always factor out the Greatest Common Factor first.
- If there are two terms, look for the difference of two squares or the difference and/or sum of two cubes.
- Factor any polynomials that have four terms.
- Factor any trinomials that can be factored.

Factoring a Trinomial of the Form $x^2 + bx + c$

The factored form of $x^2 + bx + c$ is

The product of these numbers is c.

$$x^2 + bx + c = (x + \square)(x + \square)$$

The sum of these numbers is b.

Image from Martin-Gay, E. (2009). Beginning algebra (5th). Upper Saddle River, NJ.

Multiplying Rational Expressions

Completely factor the numerators and denominators in all rational expressions.

Cancel out any like terms. Be careful when cancelling like terms. Binomials can only cancel out other like binomials. You cannot cancel out parts of a binomial. It's the whole binomial or nothing!

Write the answer in factored form. Do not multiply the terms back together.

Example

$$\frac{x^2+x}{3x} \bullet \frac{6}{5x+5}$$

$$\frac{x(x+1)}{3x} \bullet \frac{6}{5(x+5)}$$

$$\frac{(x+1)}{1} \bullet \frac{2}{5(x+5)}$$

$$\frac{2(x+1)}{5(x+5)}$$

The first step I took was to factor any terms that could be factored.

The like terms are highlighted in red and in blue. These are the terms that can be reduced or cancelled out.

Gather the remaining terms by putting them together as one term but do not multiply them together.

Example

$$\frac{x}{2x-14} \bullet \frac{x^2-7x}{5}$$

This will sound redundant but factor first!

$$\frac{x}{2(x-7)} \bullet \frac{x(x-7)}{5}$$

The common factors are highlighted in red. These are the same identical factors so we can cancel them out.

$$\frac{x^2}{10}$$

Simplify your answer. In this case, you will multiply the terms together as it is appropriate for this problem.

Example

$$\frac{(m+n)^2}{m-n} \bullet \frac{m}{m^2+mn}$$

Guess what we did first?! We factored everything that could be factored.

$$\frac{(m+n)(m+n)}{m-n} \bullet \frac{m}{m(m+n)}$$

I highlighted the like terms so I can see what can be cancelled.

Write the remaining terms

$$\frac{m+n}{m-n}$$

Practice Problems

Try the work and then check your answers on the next few pages.

$$\frac{x^2+x}{3x}\bullet\frac{6}{5x+5}$$ Answer

$$\frac{x^2+6x+8}{x^2+x-20}\bullet\frac{x^2+2x-15}{x^2+8x+16}$$ Answer

$$\frac{x}{2x-14}\bullet\frac{x^2-7x}{5}$$ Answer

$$\frac{a^2-4a+4}{a^2-4}\bullet\frac{a+3}{a-2}$$ Answer

$$\frac{(m+n)^2}{m-n}\bullet\frac{m}{m^2+mn}$$ Answer

$$\frac{x^2+9x+20}{x^2-15x+44}\bullet\frac{x^2-11x+28}{x^2+12+35}$$ Answer

Practice Problems

$$\frac{x^2+x}{3x} \bullet \frac{6}{5x+5}$$

Factor and cancel like terms!

$$\frac{x(x+1)}{\cancel{3}x} \bullet \frac{\cancel{6}^{2}}{5(x+1)}$$

$$\frac{2}{5}$$

[Return to Practice Problems]

Practice Problems

$$\frac{x}{2x-14} \bullet \frac{x^2-7x}{5}$$

$$\frac{x}{2(x-7)} \bullet \frac{x(x-7)}{5}$$

$$\frac{x^2}{10}$$

Return to Practice Problems

Practice Problems

$$\frac{(m+n)^2}{m-n} \bullet \frac{m}{m^2+mn}$$

$$\frac{(m+n)(m+n)}{m-n} \bullet \frac{m}{m(m+n)}$$

$$\frac{m+n}{m-n}$$

Return to Practice Problems

Practice Problems

$$\frac{x^2+6x+8}{x^2+x-20}\bullet\frac{x^2+2x-15}{x^2+8x+16}$$

$$\frac{\color{red}{(x+4)}(x+2)}{\color{blue}{(x+5)}(x-4)}\bullet\frac{\color{blue}{(x+5)}(x-3)}{\color{red}{(x+4)}(x+4)}$$

$$\frac{(x+2)(x-3)}{(x-4)(x+4)}$$

Return to Practice Problems

Practice Problems

$$\frac{a^2-4a+4}{a^2-4}\bullet\frac{a+3}{a-2}$$

$$\frac{(a-2)(a-2)}{(a-2)(a+2)}\bullet\frac{a+3}{a-2}$$

$$\frac{a+3}{a+2}$$

Return to Practice Problems

Practice Problems

$$\frac{x^2+9x+20}{x^2-15x+44}\bullet\frac{x^2-11x+28}{x^2+12+35}$$

$$\frac{\color{red}{(x+5)}(x+4)}{(x-11)\color{blue}{(x-4)}}\bullet\frac{(x-7)\color{blue}{(x-4)}}{(x+7)\color{red}{(x+5)}}$$

$$\frac{(x+4)(x-7)}{(x-11)(x+7)}$$

Return to Practice Problems