Solving Systems of Linear Equations

A **system of linear** equations consists of two or more linear equations.

A **solution** of a system of two equations in two variables is an ordered pair (x, y) that makes both equations true.

A system of equations with at least one solution is a **consistent system**. *A* system that has no solution is an **inconsistent system**.

If the graphs of two linear equations are identical, the equations are **dependent**.

If the graphs are" different, the equations are **independent**.

To solve a system of linear equations by the **substitution method**:

1. Solve on equation for a variable.
2. Substitute the expression for the variable into the other equation.
3. Solve the equation from step 2 to find the value of one variable.
4. Substitute the value from step 3 in either original equation to find the value of the other variable.
5. Check the solution in both equations.

To solve a system of linear equations by the **elimination method**:

1. Rewrite each equation in standard form Ax + By = *C.*
2. Multiply one or both equations by a nonzero number so that the coefficients of a variable are opposites.
3. Add the equations.
4. Find the value of one variable by solving the resulting equation.
5. Substitute the value from step 4 into either original equation to find the value of the other variable.
6. Check the solution in both equations.

A solution of an equation in three variables x, y and z is an ordered triple (x, y, z) that makes the equation a true statement.

To solve a system of three linear equations by the elimination method:

1. Write each equation in standard form, Ax + By + Cz = D.
2. Choose a pair of equations and use the equations to eliminate the same variable.
3. Choose any other pair of equations and eliminate the same variable.
4. Solve the system of two equations in two variables from steps 1 and 2.
5. Solve for the third variable by substituting the values of the variables from step 4 into any of the original equations.

A **matrix** is a rectangular array of numbers.

The **augmented matrix of the system** is obtained by writing a matrix composed of the coefficients of the variables and the constants of the system.

The following **row operations** can be performed on matrices, and the result is an equivalent matrix.

Elementary row operations:

1. Interchange any two rows.
2. Multiply (or divide) the elements of one row by the same nonzero numbers.
3. Multiply (or divide) the elements of one row by the same nonzero number and add to its corresponding elements in any other row.

***A* square matrix** is a matrix with the same number of rows and columns.

***A* determinant** is a real number associated with a square matrix. To denote the determinant, place vertical bars about the array of numbers.

The determinant of a 2 x 2 matrix is

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

Cramer's rule for two linear equations in two variables.

The solution of the system

$$ax + by = h$$
$$cx + dy = k$$

Is given by:

$$x = \frac{\begin{bmatrix} h & b \\ k & d \end{bmatrix}}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \frac{D_x}{D}$$

$$y = \frac{\begin{bmatrix} a & h \\ c & k \end{bmatrix}}{\begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \frac{D_y}{D}$$

as long as $D = ad - bc$ is not 0.

Determinant of a 3 x 3 matrix:

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} =$$

a_1 times $\begin{bmatrix} b_2 & c_2 \\ b_3 & c_3 \end{bmatrix}$ minus

a_2 times $\begin{bmatrix} b_1 & c_1 \\ b_3 & c_3 \end{bmatrix}$ plus

a_3 times $\begin{bmatrix} b_1 & c_1 \\ b_2 & c_2 \end{bmatrix}$ equals

Each 2 x 2 matrix above is called a **minor.**

Cramer’s Rule for Three Equations in Three Variables

The solution of the system

$$\begin{cases} a_1x + b_1y + c_1z = k_1 \\ a_2x + b_2y + c_2z = k_2 \\ a_2x + b_3y + c_3z = k_3 \end{cases}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & b_3 \end{vmatrix}$$

$$D_x = \begin{vmatrix} k_1 & b_1 & c_1 \\ k_2 & b_2 & c_2 \\ k_3 & b_3 & b_3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} a_1 & k_1 & c_1 \\ a_2 & k_2 & c_2 \\ a_3 & k_3 & b_3 \end{vmatrix}$$

$$D_z = \begin{vmatrix} a_1 & b_1 & k_1 \\ a_2 & b_2 & k_2 \\ a_3 & b_3 & k_3 \end{vmatrix}$$ as long as D is not 0.