Solving Systems of Equations

Substitution and Elimination Methods

Substitution Method

- The substitution method is done exactly as its name implies: substitute one equation into the other to solve for a variable.

When one equation is already solved for x or y it makes for an easy substitution problem.

Solve the following system of equations:

$$x = 2y + 7$$
$$2x + 3y = 9$$

$$2(2y + 7) + 3y = 9$$
$$4y + 14 + 3y = 9$$
$$7y + 14 = 9$$
$$7y = -5$$
$$y = -\frac{5}{7}$$

$$x = 2\left(-\frac{5}{7}\right) + 7$$
$$x = -\frac{10}{7} + 7$$
$$x = \frac{39}{7}$$

Remember that once you solve for one variable you need to substitute it back into one of the original equations to find the other variable. Your answer should be a coordinate: $\left(\frac{39}{7}, -\frac{5}{7}\right)$

When neither equation is solved for x or y we simply rearrange the equation to get one by itself.

Solve the following system of equations:

$$x - 2y = 14$$
$$x + 3y = 9$$

First thing we need to do is to solve one of the equations so that we get x or y by itself. It doesn't matter which variable you choose to solve for or which equation to use. Just pick one! I am going to choose the first equation.

Solve the following system of equations:

$$x - 2y = 14 \Rightarrow x = 2y + 14$$

$$x + 3y = 9$$

Now I can substitute in the value of x into the second equation and solve the system. Never substitute an equation that you solved for a variable back into itself. Choose the other equation.

$$(2y + 14) + 3y = 9$$

$$2y + 14 + 3y = 9$$

$$5y + 14 = 9$$

$$5y = -5$$

$$y = -1$$

Remember to substitute the variable back into one of the original equations to get the other part of the ordered pair.

$$x + 3(-1) = 9$$

$$x - 3 = 9$$

$$x = 12$$

The solution to the system is (-1, 12). Don't forget to check your answer just in case you made a mistake.

Checking

$$x - 2y = 14$$
$$x + 3y = 9$$

Solution found was (-1, 12).

$$x - 2y = 14$$
$$12 - 2(-1) = 14$$
$$12 + 2 = 14$$
$$14 = 14$$

Check the answers in both equations!

$$x + 3y = 9$$
$$12 + 3(-1) = 9$$
$$12 - 3 = 9$$
$$9 = 9$$

It works in both so you have your answer.

Elimination Method

The other method that we are going to work with is the Elimination method. Some of you may know it by the Addition method. Elimination/Addition are the same thing.

To use this method, we need to add the two equations together so that one of the variables drops out. Sometimes we can do that easily and other times it takes additional work.

First we will look at the easy one.

Solve the system of equations:

$$4x + 5y = 14$$
$$-4x - 3y = -10$$

Add the two equations together and one of the variables will drop out.

$$\begin{array}{r} \cancel{4}\,\cancel{x} + 5y = 14 \\ -\cancel{4}\,\cancel{x} - 3y = -10 \\ \hline 2y = 4 \\ y = 2 \end{array}$$

Now substitute what you found for the y value back into one of the original equations to find x.

$$4x + 5(2) = 14$$
$$4x + 10 = 14$$
$$4x = 4$$
$$x = 1$$

Solution: (1, 2)

Don't forget to check your answer!

What happens if you add the two equations together and neither of the variables drops out? Good question! That just means we have to add an extra step to the solving process.

Solve the system of equations:

$$3x + 5y = 12$$
$$4x - 3y = -13$$

$$\begin{array}{r} 3x + 5y = 12 \\ \underline{4x - 3y = -13} \\ 7x + 2y = -1 \end{array}$$

When I add the two equations together, nothing drops out. This means I have to "force" one to drop out.

$$3x+5y=12$$
$$4x-3y=-13$$

I am going to multiply the entire first equation by 4 and the second equation by -3. This means that the x values will drop out.

$$3x+5y=12 \Rightarrow 4(3x+5y=12) \Rightarrow 12x+20y=48$$
$$4x-3y=-13 \Rightarrow -3(4x-3y=-13) \Rightarrow -12x+9y=39$$

$$12x+20y=48$$
$$-12x+9y=39$$

Now when I add them together the x variable will drop out.

$$12x + 20y = 48$$
$$\underline{-12x + 9y = 39}$$
$$29y = 87$$
$$y = 3$$

$$12x + 20y = 48$$
$$12x + 20(3) = 48$$
$$12x + 60 = 48$$
$$12x = -12$$
$$x = -1$$

Solution is (-1, 3)