

## Square Roots

The square root of a number is a value that, when multiplied by itself, gives the number.

$$\text{index} \sqrt{\text{radicand}} \quad \sqrt{\quad} \text{ radical sign}$$

Examples of the first 12 square roots and cube roots (they are endless):

Square Roots				Cube Roots			
1x1	1	$\sqrt{1}$	1	1x1x1	1	$\sqrt[3]{1}$	1
2x2	4	$\sqrt{4}$	2	2x2x2	8	$\sqrt[3]{8}$	2
3x3	9	$\sqrt{9}$	3	3x3x3	27	$\sqrt[3]{27}$	3
4x4	16	$\sqrt{16}$	4	4x4x4	64	$\sqrt[3]{64}$	4
5x5	25	$\sqrt{25}$	5	5x5x5	125	$\sqrt[3]{125}$	5
6x6	36	$\sqrt{36}$	6	6x6x6	216	$\sqrt[3]{216}$	6
7x7	49	$\sqrt{49}$	7	7x7x7	343	$\sqrt[3]{343}$	7
8x8	64	$\sqrt{64}$	8	8x8x8	512	$\sqrt[3]{512}$	8
9x9	81	$\sqrt{81}$	9	9x9x9	729	$\sqrt[3]{729}$	9
10x10	100	$\sqrt{100}$	10	10x10x10	1000	$\sqrt[3]{1000}$	10
11x11	121	$\sqrt{121}$	11	11x11x11	1331	$\sqrt[3]{1331}$	11
12x12	144	$\sqrt{144}$	12	12x12x12	1728	$\sqrt[3]{1728}$	12

There are many different roots, 4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup>, etc. They all follow the same rules.

Many times you are asked to find the square root of a number without using a calculator. That means an answer will not contain a decimal answer. To find the square root without using a calculator, try to break the number down into a perfect square and another number. The “perfect square” comes out and the other number stays under the radical sign.

$$\sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$$

$$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2}$$

The “perfect square” is brought outside the radical and the rest of the number stays under the radical sign because it is not a perfect square.

$$\sqrt{\frac{1}{100}} = \frac{1}{10}$$

$$\sqrt{\frac{4}{81}} = \frac{2}{9}$$

$$\sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\sqrt{\frac{1}{121}} = \frac{1}{11}$$

Cube roots – think of three of the same numbers, when multiplied together, give you the number.

$$\sqrt[3]{8} = \sqrt{2 \cdot 2 \cdot 2} = 2$$

$$\sqrt[3]{27} = \sqrt{3 \cdot 3 \cdot 3} = 3$$

$$\sqrt[3]{-8} = \sqrt{-2 \cdot -2 \cdot -2} = -2$$

## Square Roots containing Variables and Exponents

When a square root contains variables with exponents, divide the exponent by the index to find out what you can bring out. Any remainder that is left stays under the radical sign.

$$\begin{aligned}\sqrt{x^4} &= x^2 \\ \sqrt{x^5 y^4} &= x^2 y^2 \sqrt{x} \\ \sqrt[3]{8x^6 y^8} &= 2x^2 y^2 \sqrt{y^2}\end{aligned}$$

If the index of the square root is 2, divide 2 into the exponents. If the index of the square root is 3, divide 3 into the exponents. Any remainders stay under the radical sign.

### Multiplying Square Roots

You can only multiple the outside numbers together and the numbers under the radical sign together; never the outside number with a number under the radical sign.

$$\begin{aligned}\sqrt{7} \sqrt{3} &= \sqrt{21} \\ 3\sqrt{5} \ 4\sqrt{3} &= 12\sqrt{15} \\ 2\sqrt{3} \ 3\sqrt{5} - 5\sqrt{7} &= 6\sqrt{15} - 10\sqrt{21}\end{aligned}$$

Using Foil to Multiply Square Roots: Multiply the outside numbers and the square root numbers. After multiplying, reduce your square roots, remembering to multiply the square root you are taking out with whatever is already outside the square root. Add any like terms.

$$\begin{aligned}3\sqrt{5} + 2\sqrt{3} \quad 4\sqrt{5} - 3\sqrt{9} \\ 12\sqrt{25} - 9\sqrt{45} + 8\sqrt{15} - 6\sqrt{27} \\ 12\sqrt{5 \cdot 5} - 9\sqrt{9 \cdot 5} + 8\sqrt{15} - 6\sqrt{9 \cdot 3} \\ 12(5) - 9(3)\sqrt{5} + 8\sqrt{15} - 6(3)\sqrt{3} \\ 60 - 27\sqrt{5} + 8\sqrt{15} - 18\sqrt{3}\end{aligned}$$

1. Multiply the two binomials using FOIL.
2. Determine what terms are perfect squares.
3. Multiply the perfect square by any number that is outside the radical.
4. Simplify by combining any like terms.

If you do not know what the square root of a number is, you can use the prime factorization method to find the square root.

$140 = 2 \times 2 \times 5 \times 7$       The square root is  $2\sqrt{35}$ . The 2s show up twice so we can bring out a 2. The rest is multiplied together and stays under the radical sign.

$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$       The square root is  $4\sqrt{15}$ . We have two pairs of 2s, so we bring each one out one time and multiply them together. The remainder stays under the radical sign.