

Linear Programming Problem

Setting Up System of Problem Constraint Equations

Let's take a closer look at determining a standard maximization problem and the associated system of problem constraint equations.

Suppose that the system of problem constraint equations has 16 variables, including 9 slack variables. We need to determine the following:

1. The number of constraint equations in the system.
2. The number of decision variables in the system.
3. The number of basic variables and non-basic variables associated with the system.
4. The number of linear equations and the number of variables in the system formed by setting the non-basic variables equal to 0.

That sounds like a tall order but here goes!

1. A standard maximization problem has problem constraints in the form

$$a_1x_1 + a_2x_2 + \dots + a_nx_n \leq b, \text{ where } b \geq 0$$

In order to convert these problem constraint inequalities to a system of linear equations, a slack variable is added to the left side of each inequality to write it as an equation.

The given system of problem constraint equations has 9 slack variables. Each slack variable is used to write one problem constraint inequality as an equation.

For this system, it means we must have 9 constraint equations in the system.

2. If the system has 16 variables and 9 of those are slack variables, the remaining variables are decision variables. So $16 - 9 = 7$; there are 7 decision variables in this system.
3. We will divide these variables into two mutually exclusive groups. Basic variables are selected arbitrarily with the one restriction that there are as many basic variables as there are equations in the system. Any remaining variables are called non-basic variables. Since the given system has 9 problem constraint equations, there are 9 basic variables associated with the system. Of the 16 variables in the given system, the remaining 7 are non-basic variables.
4. To find the basic solution to the system, the non-basic variables are set equal to 0 and the resulting system of equations is solved. Setting the non-basic variables

equal to 0 does not affect the number of constraint equations in the system. The resulting system has 9 linear equations. When the 7 non-basic variables are set equal to 0, the 9 basic variables remain in the resulting system of equations.

So what does all of that mean? It means that a system of problem constraint equations that has 16 variables, including 9 slack variables, has 9 equations and 7 decision variables. Associated with the system are 9 basic variables and 7 non-basic variables. When the non-basic variables are set equal to 0 to find the basic solutions to the system, the result is a system of 9 equations in 9 variables.